Application of Random Data Techniques to Aircraft Inlet Diagnostics

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The methods and nomenclature of random data analysis as applicable to inlet diagnostics are explained and defined. The pertinent statistical functions are approached from the viewpoint of one unfamiliar with these methods of analysis. Examples of the application of these techniques to scale model inlet tests are given. It is shown that these methods yield a physical description of the flowfield which could not be obtained by other means.

Introduction

THE statistical methods used in the analysis of random data are mathematical tools which are finding wide use in the aerospace sciences. They have been applied with notable success to the fluctuating pressure-panel flutter problem, and have long been used in the study of turbulence. However, these methods and the associated terminology still remain somewhat foreign to most wind-tunnel testing of scale model inlets. This may have been justified in the past, where the measurement and analysis of the steady-state pressure field in the inlet were sufficient to specify the inlet performance. However, with the advent of high-performance aircraft utilizing an augmented turbofan propulsion unit, the airframe engine compatibility has been demonstrated to depend upon the unsteady pressure field in the inlet as well as the steady-state pressure distribution.

Evaluation of the compatibility relations, instant by instant, in a quasi-steady fashion can relate the relative compatibility of the compressor face pressure field and the engine, but this does not help to explain to the inlet designer what is physically causing a particular pressure distribution. It is here that statistics can play a very useful role.

The purpose of this paper is to define and develop the functional relations and nomenclature applicable to inlet diagnostics. These methods have been applied with much success to varying inlet designs, and have been applied to entire wind tunnels. Examples will be discussed from this background to illustrate the physical implications of the statistical parameters.

Data Classification

Data can be divided into two broad categories, deterministic and random (Fig. 1). Deterministic data has the property that it can be described by an explicit mathematical equation. If, for example, $F(t)$ time histories for N runs or tests under identical conditions behave in the same manner (some differences in data traces can be seen, due to instrumentation uncertainties and test condition repeatability limitations), it is clear that under a given test condition the physical process $F(t)$ has only one outcome at a given t, ergo, these data could be curve fit and described by a mathematical equation and therefore are deterministic data.

Random data cannot be described by an explicit mathematical relation. Each run or test yields a different time history trace. At any given time t, there are an infinite number of equally likely values of Y. Therefore, no functional relation can be written for the time history traces.

The broad classification of random data is subdivided into two categories, stationary and nonstationary. Nonstationary is a term applied to random data to indicate that the statistical properties of the data vary with time so that analysis must proceed by the use of instantaneous ensemble averages.

An ensemble is a large group of separate tests or runs, and an instantaneous ensemble average is therefore the average over a large collection of runs at the same instant of time in all runs. As an example, consider the problem of finding the mean value of the nonstationary random process (Y) shown in Fig. 2. Since the data is nonstationary, the mean value of Y will be a function of time, and will have to be computed at each instant of time which is of interest. Looking at Fig. 2, the mean value by ensemble averaging at time t_1 is equal to $\langle Y(t_1)\rangle = (1/N)[Y_1(t_1) + Y_2(t_1) + Y_3(t_1) + \ldots Y_N(t_1)]$ or in summation notation

$$\langle Y(t_1)\rangle = \frac{1}{N}\sum_{i=1}^{N} Y_i(t_1) \tag{1}$$

where N is the number of test runs at identical test conditions which make up the ensemble. If the mean value over a given time interval is desired, the Y must be computed at each instant.

Another way to say the same thing may be helpful. A nonstationary process has statistical quantities which must be computed from a large collection of identical test runs, and which vary as a function of the elapsed time during each run.

Pure nonstationary processes require an ensemble averaging technique, as demonstrated for the mean value of (Y). For this reason these processes are very difficult and expensive to analyze since they require a statistically significant number of runs to construct the ensemble. In terms of wind-tunnel testing it might take several hundred runs at identical run conditions to construct a meaningful ensemble. The expense of this is obviously prohibitive. However, some nonstationary random processes can be classified into special categories which simplify the measurement and analysis problems. An example would be a nonstationary process that is represented by an ensemble in which each run is related to a single run by a functional relation. In this case, one run can be taken, and the remainder of the necessary ensemble could then be generated from this run mathematically.[1]

A stationary random process has statistical properties which are independent of time. If the ensemble for Y in Fig. 2 is now taken as stationary, the mean value of Y computed from an ensemble average of t_1 will be equal to the value computed at any other time. That is,

$$\langle Y(t_1)\rangle = \langle Y(t_2)\rangle = \langle Y(t_3)\rangle = \langle Y\rangle$$

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and

$$\langle Y \rangle = \frac{1}{N} \sum_{i=1}^{N} Y_i(t) \quad (2)$$

for any t.

This still involves an ensemble average and is characteristic of nonergodic random data. Nonergodic means that although the statistical properties computed from the ensemble are stationary or steady state, the properties computed from a single run do not sufficiently agree with values from other individual runs so that one run can be accepted as typical of the process. Therefore, the averaging over many runs at similar conditions (an ensemble) is necessary.

An ergodic process is a stationary process in which the statistical properties of each run in the ensemble are equal. In this case one run provides statistical information which accurately describes the whole process. It must be noted that this does not mean the actual data traces will look alike from run to run, but that the statistical properties will remain constant from run to run. If the ensemble in Fig. 2 is now taken to represent an ergodic process, any run in the ensemble can be chosen at random and analyzed alone to represent the process Y. Cumbersome ensemble averaging techniques are not necessary. The mean value in this case will be equal to the average value of the data over time T in one run

$$\langle Y \rangle = \frac{1}{T} \int_0^T Y dt \quad (3)$$

where T is the data time, or the total length of time taken by the run or test in question. T must be large enough to contain a statistically significant amount of data.

The ergodic process clearly represents an important class of random data because the nature of the process can be completely determined from one data sample, or one run. Fortunately, most inlet pressure data of interest fall into this group.

The classification of random data can be broken into more discrete divisions, but those defined here should be sufficient for most inlet applications.

Statistical Description of Random Data

Four basic types of statistical functions are used in wind-tunnel and/or inlet diagnostics.[2] There are 1) mean square values or rms values; 2) correlation functions; 3) spectral density functions; 4) probability density functions.

Mean Square Values

Mathematically speaking, the mean-square value of a random process (Y) is simply the average of the square values of the time history. That is,

$$\langle Y^2 \rangle = \frac{1}{T} \int_0^T Y^2 dt \quad (4)$$

The rms value is defined as a positive root of the mean square value

$$T_{\text{rms}} = \left[\frac{1}{T} \int_0^T Y^2 dt \right]^{1/2} \quad (5)$$

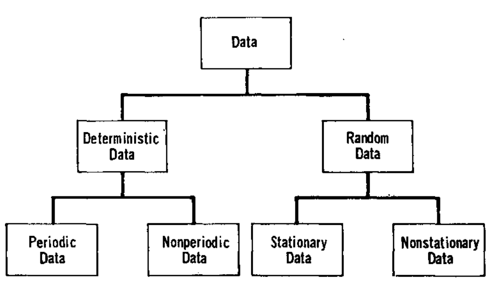

Fig. 1 Data classification.

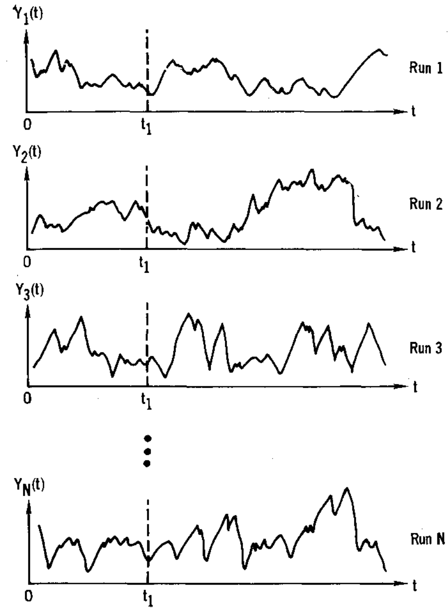

Fig. 2 Ensemble process for nonstationary data.

The equations can be applied directly to an ergodic process. For a nonergodic stationary process, ensemble averages must be used so that $\langle Y^2 \rangle$ and Y_{rms} are obtained from

$$\langle Y^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} [Y_1(t)]^2 \quad (6)$$

and

$$Y_{\text{rms}} = \frac{1}{N} \left[\sum_{i=1}^{N} [Y_1(t)]^2 \right]^{1/2} \quad (7)$$

in the same way that the mean value is obtained from an ensemble average.

If the process Y is nonstationary, the mean square or rms will have a different value at each increment of time. If the time increment is specified as t_j, $j = 1, 2, 3, \ldots$, and the run number in the collection of runs at identical conditions composing the ensemble is specified as i, then $\langle Y^2 \rangle$ and Y_{rms} can be obtained from

$$\langle Y^2(t_j) \rangle = \frac{1}{N} \sum_{i=1}^{N} [Y_i(t_j)]^2; \quad j = 1, 2, 3 \ldots \quad (8)$$

and

$$Y_{\text{rms}}(t_j) = \left[\frac{1}{N} \sum_{i=1}^{N} [Y_i(t_j)]^2 \right]^{1/2}; \quad j = 1, 2, 3 \ldots \quad (9)$$

An elementary statistical relation exists between the mean value of a process, the rms value and the standard deviation. This relationship is

$$\sigma^2 = (\text{rms})^2 - (\text{mean})^2 \quad (10)$$

where σ is the standard deviation, and σ^2 is the variance.

An example of the application of Eq. (10) is the dynamic pressure measuring system commonly employed in a pressure probe at the compressor face of an inlet duct. As seen in Fig. 3, the true time variant (dynamic) pressure can be separated into the mean, or average value, and a component which fluctuates about that mean value. This can be accomplished by using a pneumatic filter to average the dynamic pressure from a pitot tube and pass this mean, steady state value to a measuring transducer where P_s is recorded, and also pass it to the reference side of a high-response gage so that the output of the high-response transducer is essentially $P - P_s$ or P_f. This P_f oscillates about P_s, which has been sub-

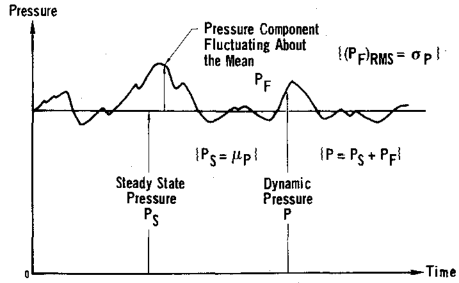

Fig. 3 Dynamic pressure.

tracted from P so that the output actually oscillates about zero. Letting the mean in Eq. (10) equal zero, it is easily seen that ΔP_{rms}, or $(P_f)_{rms}$, and σ_p, the standard deviation of the entire dynamic pressure P, are equivalent. This is the basis for the often-used relation that the peak fluctuating pressure is three times the rms value of the fluctuating pressure. What is really being applied is the relationship between standard deviation and the Gaussian probability density function for a random variable (fluctuating pressure) which has a mathematical expectation, that is to say, a mean value, of zero. This commonly used relation between ΔP_{rms} and $(\Delta P_{rms})_{peak}$ is valid only so long as the mean value of the fluctuating component is really zero, and so long as the probability density is actually very close to Gaussian.

Correlation functions

Correlation functions provide information concerning the time-wise structure of the random data. There are two divisions of correlation functions, autocorrelation and cross-correlation. Autocorrelation functions describe the relationship between data at a given time t and data at a later time $(t + \tau)$ where τ is the lag time.

Y represents a random stationary ergodic process. An indication of the correlation between a data point at t and a point at a time τ later can be obtained by simply multiplying together the values at the two times. This product represents an indication of the general agreement of the two points separated in time by the amount τ. To obtain a statistically meaningful correlation, this product must be averaged over a statistically significant number of points. For ergodic data Y this can be accomplished by integrating the product $Y(t)Y(t + \tau)$ over a sufficiently long data time (T)

$$R_y(\tau) = \frac{1}{T}\int_0^T Y(t)\cdot Y(t+\tau)dt \qquad (11)$$

If the process is nonergodic and stationary, an ensemble average would be necessary to obtain

$$R_y(\tau) = \frac{1}{N}\sum_{i=1}^{N} Y_i(t)\cdot Y_i(t+\tau) \qquad (12)$$

and if the process were nonstationary, the autocorrelation function $R_y(\tau)$ would also be a function of the time chosen for the initial data points. In this case

$$R_y(t_j,\tau) = \frac{1}{N}\sum_{i=1}^{N} Y_i(t_j)\cdot Y_i(t_j+\tau); \quad j = 1, 2, 3 \ldots \qquad (13)$$

where N is the number of test runs in the ensemble.

From the definition of mean-square values, it is obvious that if τ is set equal to zero in Eqs. (11–13) they become identical to Eqs. (4, 6, and 8). Therefore, a fundamental relationship exists between the mean square or rms value and the autocorrelation function for a given random set of data. Explicitly, the mean square value is identical to the autocorrelation function evaluated at a lag time of zero. Mathematically this relation may be written as

$$Y_{rms} = [R_y(0)]^{1/2} \qquad (14)$$

or

$$\langle Y^2\rangle = R_y(0) \qquad (15)$$

The autocorrelation function always has its maximum value at $\tau = 0$, and as the lag time increases, can be either positive or negative.

Another relationship, not quite so apparent, is that the autocorrelation function for large values of lag time approaches the mean value of the data squared, so long as the data are random in nature. This can be expressed as

$$\langle Y\rangle = [R_y(\infty)]^{1/2} \qquad (16)$$

or

$$\langle Y\rangle^2 = R_y(\infty) \qquad (17)$$

This means that the autocorrelogram [plot of $R(\tau)$] of random data must start at zero lag time with a value equal to the mean square of the data and decrease in amplitude until at some lag time τ it reaches the value of the mean of the data, squared.

The technique of autocorrelation can also be applied to deterministic data. The result is that the autocorrelation does not decay to zero (assuming $\langle Y\rangle = 0$) but persists for all lag times. The correlogram of deterministic data buried in random noise is equal to the simple superposition of the autocorrelograms of the random noise and the deterministic data taken separately.[1] This makes autocorrelation a powerful tool for the detection of sinusoids or other deterministic components masked by the random fluctuating pressure.

For inlet testing, normal practice has been to mount the inlet configuration on a similarly scaled parent fuselage to duplicate the appropriate flowfield at the inlet entrance. This practice affords a convenient mounting position (the fuselage nose) for a fluctuating pressure transducer to monitor the freestream wind-tunnel noise. Any unusual perturbation occurring in a wind-tunnel facility will be detected by the nose-mounted transducer. This allows the identification of perturbations in the inlet which are caused by freestream effects.

The technique of autocorrelation can be applied to the freestream fluctuating pressure, as measured by the nose probe, and also to probes in the inlet. The purpose is twofold. First, and most important, is the detection of periodic and/or deterministic components in the pressure data. The random components will, with increasing lag time, approach zero, thereby exposing the deterministic components. If the fluctuating pressure is separated by subtracting the mean pressure at the high-response transducer, the mean value of the fluctuating signal will be zero. Thus, it is expected that as lag time increases the random component of the pressure data will go to zero and any nonrandom, deterministic data component will be extracted.

The second purpose is to get a feel for the inferred scale of ideal isotropic turbulence, i. e., the length of turbulence cells passing the transducer station with velocity V. If the lag time required for the random component in the correlation to

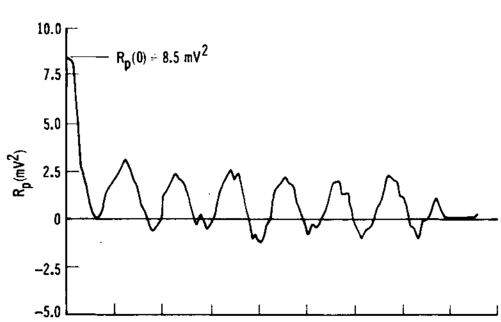

Fig. 4 Autocorrelation of freestream nose transducer output.

go to zero is τ, it can be reasoned that the characteristic structural length of turbulence passing the station of the probe is given by the simple relation

$$L^* = \tau V \qquad (18)$$

This value of L^* conveys a knowledge of the relative size of the turbulent cells with respect to inlet dimensions.

Figure 4 shows the autocorrelation function obtained from an analysis of the data from a nose transducer on a $\frac{1}{6}$-scale model inlet test in a continuous-flow wind tunnel. The autocorrelation coefficient decreases rapidly from unity to zero for the random components in about 0.06 msec. At longer lag times a nearly periodic component is extracted. This repetitive wave form has a period of about 0.143 ($\frac{1}{7}$) msec, and is therefore characterized by about a 7000-Hz frequency. The wind-tunnel freestream flow is seen to have a "noise" characteristic consisting of random turbulence and sound, and a nonrandom deterministic component which can be described as a distorted 7000-Hz sine wave. The decay time of 0.06 msec for the random component, in conjunction with the freestream velocity, which was about 2200 fps in this case, indicates that the characteristic length of turbulence entering the inlet is $L^* = 2.2(10^3)(0.06)(10^{-3})$, or 0.132 ft (a little over 1.5 in.). If you picture an ideal frozen turbulence pattern moving with velocity V down the tunnel, the length along this frozen pattern that can be traveled before all relation to the starting point is lost is L^*, or about $1\frac{1}{2}$ in. in this case.

The cross-correlation function describes the general dependence of one set of data on another set.[1] It is obtained in the same manner as the autocorrelation function, except that the product of data at t and $(t + \tau)$ is formed by using one set of data for the t values and the other set of data for the $(t + \tau)$ values. Consider two random data time histories from processes $Y(t)$ and $X(t)$. The cross-correlation function is obtained by averaging the product $Y(t)X(t + \tau)$ over some statistically significant data time T. Thus,

$$R_{yx}(\tau) = \frac{1}{T}\int_0^T Y(t)\cdot X(t + \tau)dt \qquad (19)$$

If the process is nonergodic stationary, then $R_{yx}(\tau)$ must be obtained by averaging over an ensemble (collection of runs) made up of pairs of $Y(t)$ and $X(t)$

$$R_{yx}(\tau) = \frac{1}{N}\sum_{i=1}^{N} Y_i(t)X_i(t + \tau) \qquad (20)$$

If the data considered is nonstationary, the autocorrelation function becomes a function not only of lag time τ, but also of initial time t

$$R_{yx}(t_j,\tau) = \frac{1}{N}\sum_{i=1}^{N} Y_i(t_j)X_i(t_j + \tau); \quad j = 1, 2, 3, \ldots \qquad (21)$$

It is apparent that if negative lag times are used, multiplying $Y(t)$ by $X(t - \tau)$ would give the same result as starting with t on X at $t - \tau$ and taking $X(t)Y(t + \tau)$. This is the rationale behind the statement

$$R_{yx}(-\tau) = R_{xy}(\tau) \qquad (22)$$

which is an important characteristic of the cross-correlation function. It means that the resultant function is dependent upon the order in which the lag times are taken.

Two relations which bound the cross-correlation function are

$$|R_{yx}(\tau)|^2 \leq R_x(0)R_y(0) \qquad (23)$$

and

$$|R_{yx}(\tau)| \leq \tfrac{1}{2}[R_x(0) + R_y(0)] \qquad (24)$$

As previously stated, the cross-correlation function describes the general interdependence of two sets of data. If the two sets of random data do not affect each other, that is, if they are statistically independent, then the cross-correlation of the two will be equal to the product of the mean values of the two sets of data. If the mean value of either process is zero, then the cross-correlations will be zero. Explicitly, if $X(t)$ is not being influenced by $Y(t)$, then $X(t)$ and $Y(t)$ are said to be uncorrelated, and

$$R_{yx}(\tau) = \langle Y(t)\rangle\cdot\langle X(t)\rangle \qquad (25)$$

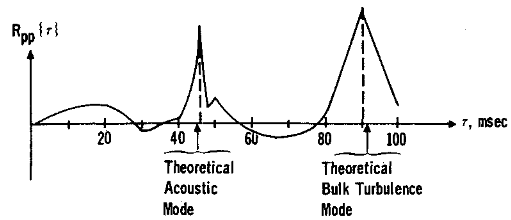

Fig. 5 The use of cross-correlation analysis in a wind tunnel.

The cross-correlation function has many uses but the most significant uses for inlet applications are the measurement of delay times, the determination of transmission paths, and the detection and recovery of deterministic signals masked by noise, where the form of the masked signal is known.

One experiment in cross-correlation analysis was completed in a blow-down wind tunnel. In this experiment the fluctuating pressures in the stilling chamber and on the test cabin wall were obtained. The purpose of the experiment was to determine if fluctuations in the stilling chamber were affecting fluctuations in the boundary layer on the test section wall. The outcome of cross-correlation of the two fluctuating pressures is shown in Fig. 5. One very narrow spike occurred at 45.5 msec, while another, broader spike is seen at 90 msec. The presence of these spikes in the cross-correlogram indicates that disturbances passing through the stilling chamber do affect the fluctuating pressures on the wall of the test cabin. The lag times associated with the peaks are equal to the transmission times of the disturbaces. The first spike indicates the presence of acoustic energy passing through the local gas between the stilling chamber and the test section. The transmission time of a sound wave was found analytically for the conditions of the run in question. The second peak indicates that bulk turbulence passes from stilling chamber to test section. This hypothesis was checked by computing the theoretical transmission time for bulk turbulence moving at the local stream velocity.

The cross-correlogram can also be used to detect a signal in a random noise background if the form of the signal is known. The technique is to tape the signal that is to be detected, and cross-correlate it with the data in which the signal is to be detected. If the signal is present, a spike will occur, and if the signal is not present, the cross-correlation will have a value equal to the product of the mean values of the signal and the data.

The cross-correlation function is a very useful tool in random data analysis. It may, however, fail to yield meaningful results if the process being studied has a transmission velocity which is strongly a function of frequency. If the phenomenon being studied propagates a fixed distance with a range of velocities dependent upon frequency, the cross-correlation will not show sharp peaks because the entire phenomenon arrives at the downstream station over a range of lag times dependent upon frequency, rather than arriving at a discrete lag time. The spectral density functions can often be employed in this case.

In the example of autocorrelation applied to the nose probe measurements of tunnel noise (Fig. 4), a 7000-Hz deterministic, periodic component was detected. The question which naturally arises is, "Does this periodic component affect the flowfield in the duct?" Even though autocorrelation of duct

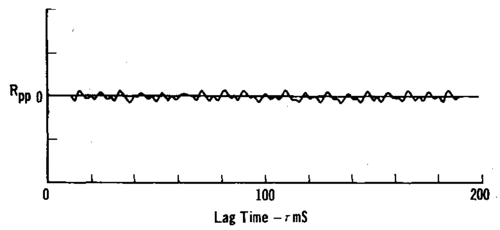

Fig. 6 Cross-correlation between freestream transducer and engine face.

fluctuating pressures does not reveal any deterministic components, except during buzz, it is desirable to have a more direct check. Cross-correlation can be applied between the nose probe and probes in the inlet-duct. The result of this cross-correlation for the case in question is shown in Fig. 6. The correlation level is zero for all lag times, indicating no significant coupling between freestream noise and the fluctuating pressures in the duct.

One very practical problem in inlet work is the determination of probe spacing. The spacing must be great enough to avoid critical blockage, and small enough to be certain that the curve fitting between probes accurately describes the instantaneous pressure distribution. Blockage consideration can be easily determined in the usual manner, but how to know that the probes are close enough to permit valid interpolation of the pressure field between them is not quite so straight forward.

Using the statistical crutch of cross-correlation analysis, an unambiguous method of determining maximum probe spacing can be determined. If pressure fluctuations at two locations in a random field are totally independent of each other such that the pressure fluctuations at one location have no bearing upon the pressure fluctuations at the other location, the cross-correlation coefficient between them, evaluated for zero lag time, will necessarily be zero. Although the reverse is not rigorously true, if the cross-correlation coefficient between two probes in an inlet is of an insignificant level, or zero, it strongly indicates that the pressures at the two probes are unrelated. That is, a knowledge of the output of one probe will not enable any extrapolation or regression to predict anything about the pressure at the other probe. Obviously, then, if a curve fit between two probes is to have any chance of providing interpolated pressures which have any bearing on reality, the correlation fields of the two probes must overlap at a statistically significant value, i.e., a value that could not have been obtained due to chance. This concept is illustrated in Fig. 7.

Spectral density functions

The power spectral density describes the frequency distribution of the mean-square values (energy density) of the data. A physical "feel" for the nature of the power spectral density function can best be obtained by considering the step-by-step procedure involved in its calculation.

Let $Y(t)$ be an ergodic random time history for the process Y. To obtain a plot of the power spectral density (power spectra) of $Y(t)$, the following method can be used: 1) Filter the data with a tuneable narrow band pass filter, with a window of B, and the filter set to some center frequency f. 2) Square the instantaneous value of the output of the band pass filter. 3) Average the squared instantaneous values over the data time T to obtain a mean square value which is located in the band width, BHz, centered at f. 4) Divide this mean square output by the band width B, to obtain one point of the power spectral density. 5) Tune the band pass filter to the next higher center frequency located BHz from the previous point, and return to step 1.

The result will be a plot of $\langle Y^2 \rangle / B$ vs f, which is the power spectral density. Note the units of power spectral density. If Y is a pressure, then the power spectral density will be in terms of the mean square pressure contained in a given band width B, divided by that band width, $\langle P^2 \rangle / B = (\text{PSI})^2/\text{Hz}$.

If $Y(t,f,B)$ represents that portion of the random data $Y(t)$ contained in a band width B centered at a frequency f, then the power spectral density $G(f)$ can be obtained from the relation (assuming an ergodic process)

$$G_y(f) = \frac{1}{BT} \int_0^T Y^2(t,f,B)dt \tag{26}$$

Once the power spectral density function has been obtained, the rms value contained in any arbitrary band of frequencies from f_1 to f_2 can be obtained simply as

$$\langle Y^2 \rangle (f_1,f_2) = \int_{f_1}^{f_2} G(f)df \tag{27}$$

or

$$Y_{\text{rms}}(f_1,f_2) = \left[\int_{f_1}^{f_2} G(f)df \right]^{1/2} \tag{28}$$

Consider, for example, a buffet test in which it is desired to determine the component of effective driving force on the model due to random pressure fluctuations at the flutter frequency of the model.

If the power spectral density of the fluctuating pressure is obtained, the effective component driving the model at its critical frequency is obtained by simply integrating over a small band width centered at the critical frequency and taking the square root. The result will be the rms pressure component in question.

From this discussion it should be apparent that if the power spectral density of a fluctuating pressure is integrated over all frequencies and raised to the one-half power, the over-all rms pressure will be obtained. In terms of an arbitrary random process $Y(t)$,

$$Y_{\text{rms}} = \left[\int_0^\infty G(f)df \right]^{1/2} \tag{29}$$

The power spectral density and the autocorrelation functions are related in an important manner. Specifically, for stationary data, they are Fourier transform pairs. That is,

$$G_y(f) = 2 \int_{-\infty}^{\infty} R_y(\tau)e^{-i\omega\tau}d\tau \tag{30}$$

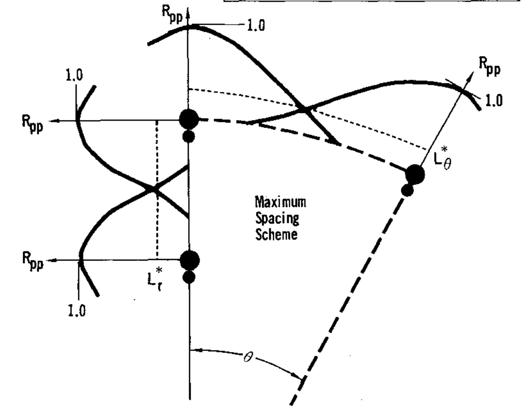

Fig. 7 Model scale and transducer spacing.

or, in more convenient form,

$$G_y(f) = 4 \int_0^\infty R_y(\tau) \cos(2\pi f \tau) d\tau \qquad (31)$$

The principal application of the power spectral density function is the determination of frequency composition as illustrated in the buffet test example. As another example, consider the narrow band power spectral density (PSD) function for the same freestream noise data used to generate the autocorrelogram in Fig. 4. Band width frequencies for this calculation were 5 Hz (10 to 50 Hz), 10 Hz (50 to 500 Hz), and 50 Hz (500 to 10^4 Hz). Figure 8 is a plot of this PSD function, revealing the tunnel noise spectrum to be characteristic of wide band random noise with too distinct spikes at 25 and 6900 Hz. The 6900-Hz spike is known from the autocorrelation in Fig. 4 to be the result of a deterministic, nearly periodic, distorted, sinusoidal component. Since the ratio of rms pressure oscillating at this frequency to the over-all rms fluctuating pressure is simply the ratio of the area under the spike to the total area under the PSD curve, it can be seen that this 6900-Hz component (determined as being about 7000 Hz by the correlation analysis) does not contain much energy.

Figure 9 shows the narrow band PSD for the compressor face fluctuating pressure for the same test conditions.

These pressures at the compressor face are wide band random data characteristic of typical inlet operation. Energy is not concentrated at any given frequency, and no deterministic or nonrandom components are visible.

It was noted that the power spectral density is actually the Fourier transform of the autocorrelation function. A cross-correlation function has also been defined. The concept of the cross-power spectral density evolves directly from the concept of the Fourier transform of the cross-correlation function. Since the cross-correlation is not an even function, the cross-power spectral density will generally be a complex number.

$$G_{yx}(f) = C_{yx}(f) - iQ_{yx}(f) \qquad (32)$$

Where $C_{yx}(f)$, the real part, is termed the "cospectral density," and $Q_{yx}(f)$, the imaginary part, is called the quadrature spectral density. The calculation of cross-power spectral density from two stationary data time histories can proceed as follows: 1) The two time histories $Y(t)$ and $X(t)$ are individually filtered by narrow band pass filters having identical band widths BHz, and the same center frequency. 2) The instantaneous values of the two filtered signals with no phase shift are multiplied. 3) The product is averaged over the data time to find the mean product of Y and X in B. 4) The mean product divided by the band width (B) represents one point at the band width center frequency for the cospectral density.

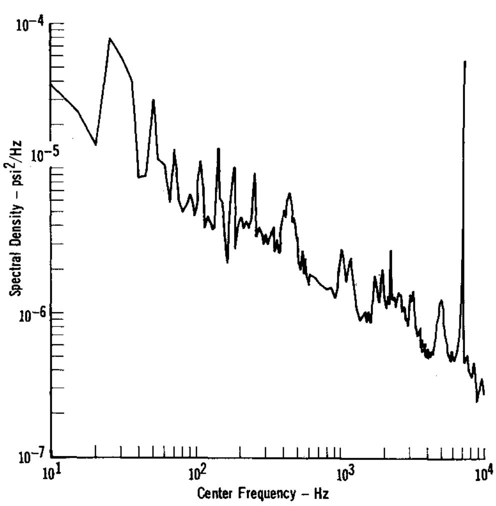

Fig. 8 Narrow band power spectral density of freestream noise.

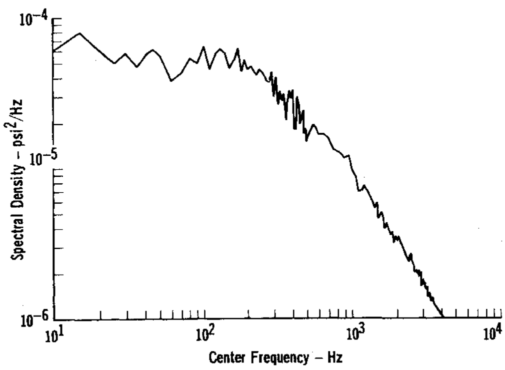

Fig. 9 Narrow band power spectral density of engine-face rake transducer output.

5) Multiply the two filtered signals from step 1 with one signal shifted 90° out of phase with the other. 6) Average the product of step 5 to obtain the mean product. 7) Divide the mean product of step 6 by the band width to obtain one point at the band width center frequency for the quadrature spectral density. 8) Step the band centers to another frequency and return to step 1.

In mathematical terms,

$$C_{yx}(f) = \frac{1}{BT} \int_0^T Y(t,f,B)X(t,f,B)dt \qquad (33)$$

$$Q_{yx}(f) = \frac{1}{BT} \int_0^T Y(t,f,B)X'(t,f,B)dt \qquad (34)$$

where the notation $Y(t,f,B)$ means that portion of $Y(t)$ which is in a band width B centered at f. The notation X' indicates that the data $X(t)$ have been shifted 90° in phase to produce X'.

Cross-power spectral density is usually written, for convenience, in complex polar notation in the form

$$G_{yx}(f) = |G_{yx}(f)| \exp[i\theta_{yx}(f)] \qquad (35)$$

where the magnitude can be found from

$$|G_{yx}(f)| = [C^2_{yx}(f) + Q^2_{yx}(f)]^{1/2} \qquad (36)$$

and the phase angle is given by

$$\theta_{yx}(f) = \tan^{-1}[Q_{yx}(f)/C_{yx}(f)] \qquad (37)$$

It can be shown that

$$C_{yx}(f) = C_{xy}(f) \qquad (38)$$

$$Q_{yx}(f) = -Q_{xy}(f) \qquad (39)$$

which implies the important relationships

$$G_{yx}(f) = G_{xy}{}^*(f) \qquad (40)$$

and

$$G_{yx}(f) = G_{xy}(-f) \qquad (41)$$

where $G_{xy}{}^*$ is the complex conjugate of $G_{xy}(f)$.

A useful limit on the magnitude of the cross-power spectral density is given by the relation

$$|G_{yx}(f)|^2 \leq G_y(f)G_x(f) \qquad (42)$$

That is, the magnitude of the cross-spectrum squared is less than or equal to the product of the individual power spectral densities.

The complex nature of the cross-power spectral density function makes it sometimes more convenient to apply a noncomplex, real valued function called the coherence function. The coherence function is obtained directly from Eq. (42) and can be written as

$$\gamma^2{}_{yx}(f) = |G_{yx}(f)|^2/G_y(f)G_x(f) \qquad (43)$$

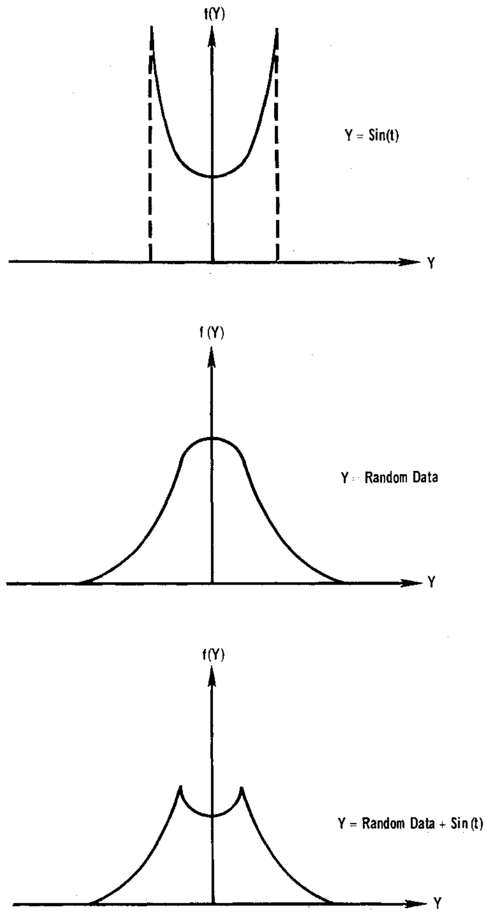

Fig. 10 Probability density plots.

From Eq. (42) it is apparent that the coherence function has a value less than or equal to unity at all times.

$$\gamma^2_{yx}(f) \leq 1.0 \quad (44)$$

Recall that the cross-correlation function for two sets of random data could be used to determine the correlation of agreement of two signals at various lag times. The coherence function in a like manner will show the correlation or agreement of two signals at various frequencies. Explicitly, if the coherence function for two sets of random data is zero at some frequency, then the two signals are incoherent or uncorrelated at that frequency. If at some other frequency the coherence is unity, then the two signals are fully coherent or fully correlated at that frequency. Thus, the coherence function can be easily used, since it has only real values between zero and unity, to illustrate the general dependence of one set of data upon another as a function of frequency.

The cross-power spectral density function has many uses, but of primary concern in gas dynamics work are the following: *A*) measurement of frequency response; *B*) measurement of time delays or transmission times as a function of frequency.

The cross-power spectral density can be used to determine both magnitude and phase of the frequency response of a system to arbitrary or random excitation. The frequency response function is defined as

$$H(f) = G_{p_1p_2}(f)/G_{p_1}(f) \quad (45)$$

That is, the frequency response is equal to the cross-power spectral density function between the system input and output, divided by the power spectral density function of the input. For example, consider the wind-tunnel test of an inlet duct model. Suppose that it is desired to determine how much noise the duct will generate due to internal flow phenomena. The problem is to determine what component of the rms noise level is really due to the duct flow and what is due to external tunnel noise. To make this determination the duct could be mounted in an anechoic chamber and bombarded with random noise. This external noise is the input. The noise measured inside the duct is the output. By obtaining the cross-power spectral density function of input to output noise, and the power spectral density function of the input noise, $H(f)$ for the duct can be calculated from Eq. (45).

The duct could then be tested in the wind tunnel. The noise in the freestream could be measured and the power spectral density obtained. With this and the frequency reponse, the component of noise inside the duct due to the external noise can be calculated as

$$G_{p_2} = |H(f)|^2 G_{p_1}(f) \quad (46)$$

where $H(f)$ is the previously determined frequency response function of the duct, and $G_{p_1}(f)$ is the measured tunnel noise power spectral density. The power spectral density generated inside the duct due to tunnel noise, $G_{p_2}(f)$, can then be integrated over all frequencies according to Eq. (29) to yield the rms pressure component in the duct due to tunnel noise. Subtracting this component from the total rms pressure measured in the duct will give the desired quantity, i.e., the noise generated in the duct by flow in the duct. In this example it is assumed that $H(f)$ is the same in a flowing and nonflowing system. This assumption would have to be verified before the example could be put to use.

It has been shown that the cross-correlation function between two random data traces will directly yield the presence of correlation between the two sets and the associated lag or transmission time. It was noted that if the phenomena propagating between two pickups had a velocity which was highly dependent on frequency, the cross-correlation function would fail to show definite peaks due to the smearing out of transmission times. It was also demonstrated that the cross-power spectral density could be used to determine correlation between two sets of data as a function of frequency by application of the coherence function. The lag of transmission times associated with the peaks of correlation for various frequencies can also be determined. The phase angle of the cross-spectral density between data sets Y and X is $\theta_{yx}(f)$. For a peak in coherence function at a given frequency, $\theta_{yx}(f)$ is the phase shift associated with the two sets. The lag time associated with this frequency is then given by

$$\tau = [\theta_{yx}(f)/2\pi f] \quad (47)$$

Recall the example of cross-correlation analysis which has been shown for fluctuations in the stilling chamber and on the test cabin wall. It was found that a very sharp spike occurred at about 45 msec, indicating the transmission of a sound field propagating from stilling chamber to test section. A second

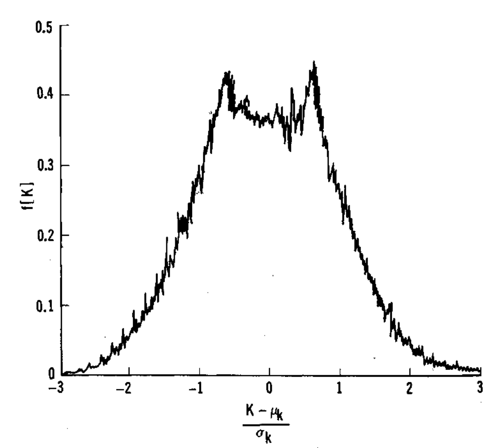

Fig. 11 Buzz onset.

spike which peaked at about 90 msec corresponded to the mean lag time for bulk transmission of turbulence. Note, however (Fig. 5), that this second peak is more spread out than the first. The transmission velocity of turbulence may be slightly dependent upon the frequency of the turbulence. It would be very desirable to determine if this is indeed the case. This could be accomplished by computing the cross-power spectral density and coherence function between the stilling chamber and the test section data. The transmission of turbulence with a distribution of frequencies will be apparent as a series of peaks in the coherence function at various frequencies. The lag times associated with these peaks can then be obtained from Eq. (47). In this way transmission time as a function of frequency can be determined.

The cross-power spectral density function is a powerful tool with a wide variety of applications to general wind-tunnel diagnostics.

Probability density functions

The probability density function describes the amplitude behavior of random data.[3,4] It indicates the probability that the data will fall in any given range of amplitudes at any instant in time. The properties of this function will be made more clear by considering the manner in which the probability density function is obtained.

Consider $Y(t)$, an ergodic random variable. 1) Filter the data with an amplitude band pass filter of window W such that $W = (Y + \Delta Y) - Y$. 2) Accumulate the total time during which the data are in the window W. 3) Divide this time in the window by the total data time to obtain the average data time during which Y has a value within W. 4) Divide the average value by the window width W to obtain a point of the probability density function at the center of the window $(Y + \Delta Y/2)$. 5) Move the window center to the next higher location and repeat the process to obtain a plot of probability density, $f(y)$, vs amplitude, $Y(t)$.

The total time during which Y is within W is

$$t_w = \Delta t_1 + \Delta t_2 + \Delta t_3 + \ldots = \Sigma \Delta t_i \quad (48)$$

If the total data time is T and the band pass filter window is W, then the probability density function is given by

$$f(y) = t_w(y)/TW \quad (49)$$

with t_w defined by Eq. (48).

Figure 10 shows typical probability density functions for a sine wave, random data, and a sine wave buried in a random noise background.

The probability density function of a sine wave has a characteristic dish shape. The random data have a distribution which is similar to a normal distribution. The probability density function of the sine wave masked by random noise

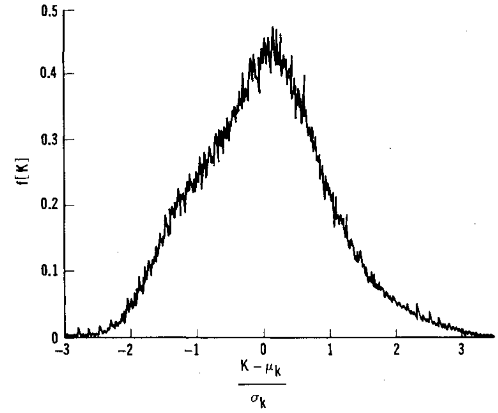

Fig. 13 Critical operation.

shows characteristics of both. This demonstrates the possible use of probability density functions to detect periodic data buried in random data. Generally speaking, the autocorrelation function provides a more powerful technique for this purpose.

The principal application of a probability density function is to establish the probabilistic description of the instantaneous amplitude value of the data. For example, consider a wind-tunnel inlet test where the pressure distortion at the inlet face is important. Suppose that for some reason it is desired to know how long the distortion maintains a value near P_1 psi. If the probability density function has been determined, this can easily be obtained.

The probability that the pressure distortion is within $\pm \Delta P$ of P, is simply the integral of f_1 from $P_1 - \Delta P$ to $P_1 + \Delta P$, where (f_1) is the probability density function of the distortion. Assume, for example, that the integral evaluated to 0.4. This means that the probability that the distortion is within $\pm \Delta P$ of P_1 is 0.4, or, in other words, it will have that value $\frac{4}{10}$ of the time.

Probability density analysis of inlet data has verified that except during buzz, the fluctuating pressure time history of any probe on the compressor face is nearly Gaussian. During buzz of the inlet a strong sinusoidal component can be detected in the data and is reflected as a cup-shaped top for the usual Gaussian distribution. As a matter of curiosity, a probability density analysis was also run on dynamic inlet engine compatibility parameters. Figures 11–14 show the results for inlet operating conditions of buzz onset, subcritical without buzz, 100% critical, and supercritical for a typical distortion parameter. The presence of the characteristic buzz frequency is seen in Fig. 11. For subcritical operation (Fig. 12) a very nearly Gaussian distribution is obtained. Peak amplitude occurs at about 3. For critical operation (Fig. 13), the distribution is skewed and the peak amplitude of the parameter occurs at about 3.5 For supercritical operation (Fig. 14), the peak value is about 5.5. This means that to predict what the peak or most critical value of distortion is

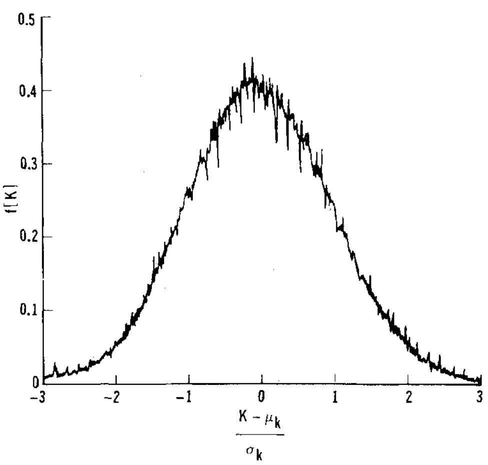

Fig. 12 Subcritical operation.

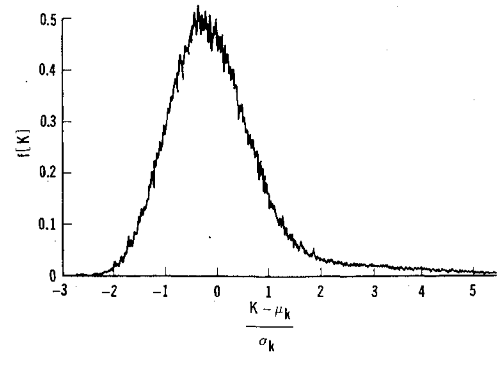

Fig. 14 Supercritical operation.

from a very short data sample, the average value should be computed, giving μ_k. Then the standard deviation σ_k should be computed. The worst case distortion parameter can then be calculated for this inlet under these flow conditions from the following relations: subcritical: $K_{\max} = 3\sigma_k + \mu_k$; critical: $K_{\max} = 3.5\sigma_k + \mu_k$; supercritical: $K_{\max} = 5.5\sigma_k + \mu_k$; where K represents this distortion parameter.

Summary

When dealing with random data, which cannot be described with explicit mathematical relations, the methods of statistical analysis can be applied to gain valuable knowledge concerning the characteristics of the data. These methods, with a long history of use in other areas, should be applicable to inlet diagnostics to a much greater degree than in the past.

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Correlation of Turbofan Engine Thrust Performance with Compound Nozzle Flow Theory

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Data from extensive altitude chamber testing of a single nozzle turbofan engine demonstrate the dependence of engine gross thrust on nozzle pressure ratio and simulated flight Mach number (or ram pressure ratio). Variations of gross thrust up to 5% at a constant nozzle pressure ratio are attributed to changes in the engine bypass ratio with power setting. A one-dimensional, compound nozzle flow theory (neglecting mixing) is employed to predict the influence of bypass ratio on the gross thrust of the dual stream nozzle flow. The results emphasize the importance of proper simulation of engine bypass ratio and nozzle pressure ratio in establishing the gross thrust coefficient for inflight thrust evaluation.

Nomenclature

A = flow area
A_{nozzle} = total nozzle flow area at given x
$A_P{}^*$ = characteristic area of primary stream, Eq. (16)
A_T = nozzle exit flow area
B = fraction of airflow bled at compressor discharge
C_G = gross thrust coefficient for single stream nozzle flow, Eq. (1)
$C_{G(2)}$ = gross thrust coefficient for dual stream nozzle flow, Eq. (22)
C = constant
F_G = gross thrust
$F_{G\text{meas}}$ = measured engine gross thrust
$F_{G\text{theo}}$ = theoretical gross thrust for single stream nozzle flow, Eqs. (2) and (3)
$F_{G\text{theo}(2)}$ = theoretical gross thrust for dual stream nozzle flow, Eq. (12)
f_1, f_2 = functional relationship
h_P = altitude
H_{TA} = total enthalpy of air
H_{TG} = total enthalpy of combustion products
HV = fuel heating value
M = Mach number
N_H = high-pressure compressor rotor speed
P_{TN} = measured nozzle inlet total pressure (see Fig. 1)
P_T = total pressure
P = static pressure
R = gas constant
T_T = total temperature
W = mass flow rate
W_A = air mass flow rate
W_{ABLD} = bleed air mass flow rate at compressor discharge
W_{AT} = total engine air mass flow rate
W_F = fuel mass flow rate
W_G = combustion products mass flow rate
X = dual stream nozzle gross thrust parameter, Eq. (13)
$\%\Delta X/\%\Delta\gamma_P$ = percent change x/percent change γ_P
β = engine bypass ratio
γ = ratio of specific heats
Φ = thrust parameter defined in Eq. (7)
ψ = nondimensional gross thrust for single stream nozzle flow, Eq. (11)
η_B = combustor efficiency
θ_T = corrected total temperature

Subscripts

$(\ )_0$ = simulated freestream flight conditions
$(\ )_{2,2F,3,4,5}$ = engine station designation (see Fig. 1)
$(\ )_e$ = exit plane of converging nozzle
$(\ )_P$ = properties of the primary (turbine discharge) stream
$(\ )_S$ = properties of the secondary (fan discharge) stream
$(\ )_X$ = axial location in the nozzle

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